In problem 1 below we analyze the strategic form of the alternating offer bargaining game, and discuss rationalizable and Nash Equilibrium (NE) strategies - with the view of having them as a reference point for comparison with subgame perfect equilibria (SPE) outcomes of the game which we will discuss in the lecture notes 3.

**Problem 1 (Strategic form of the bargaining game)**

*We consider an alternating offer bargaining game à la Rubinstein (1982). Two players negotiate on how to split a pie of size* 1*. There is infinitely many stages-opportunities to agree on the split of the pie. The game begins with period* 1*, where player* 1 *suggests that she gets a share* $x_1^1 \in [0, 1]$ *of the pie, and player* 2 *gets a share* $1 - x_1^1$*. Provided that the agreement has not been reached prior to period* $t$*, player* 1 *makes an offer* $x_1^t \in [0, 1]$ *if* $t$ *is odd, and player* 2 *makes an offer* $x_1^t \in [0, 1]$ *if* $t$ *is even. Future is discounted by a factor of* $\delta$*: if players agree on the split* $x_1^t$ *in period* $t$*, then the payoff of player* 1 *is* $\delta^{t-1}x_1^t$*, and the payoff of player* 2 *is* $\delta^{t-1}(1 - x^{t-1})$*. The payoff to never reaching an agreement is* 0 *for both players.*

*a)* Give a formal definition of a strategy for players 1 and 2

*b)* Consider the following strategy of player 2: "Regardless of the history of the game, refuse all offers but $x_1^t = 0$ in the odd periods, and always offer $x_1^t = 0$ in the even periods". Is this strategy rationalizable? Does your answer depend on $\delta$?

*c)* Can you construct Nash Equilibrium where the agreement is reached in period 100 with any division of the pie?

As opposed to NE, SPE imposes sequential rationality which makes it an attractive solution concept for multi-stage games. In problem 2 we compare NE predictions to SPE predictions of the important multi-stage game which was first analyzed in evolutionary biology to study a conflict where two players compete for an exclusive resource. In economics this game has been applied to study, among other things, *R&D* races and political lobbying.

**Problem 2 (SPE in the war of attrition)**

*Two players are fighting for a prize whose current value at any time* $t = 0, 1, 2, ...$ *is* $v > 1$. *Fighting costs* 1 *unit per period. The game ends as soon as one of the players stops fighting. If one player stops fighting in period* $t$, *he gets no prize and incurs no more costs, while his opponent wins the prize without incurring a fighting cost. If both players stop fighting at the same period, then neither of them gets the prize. The players discount their costs and payoffs with discount factor* $\delta$ *per period.*

*This is a multi-stage game with observed actions, where the action set for each player in period* $t$ *is* $A_i(t) = \{0, 1\}$, *where* 0 *means continue fighting and* 1 *means stop. A pure strategy* $s_i$ *is a mapping* $s_i : \{0, 1, ...\} \to A_i(t)$ *such that* $s_i(t)$ *descibes the action that a player takes in period* $t$ *if no player has stopped the game in periods* $0, ..., t-1$. *A behavior strategy* $b_i(t)$ *defines a probability of stopping in period* $t$ *if no player has yet stopped.*

*a)* Consider a strategy profile $s_1(t) = 1$ for all $t$ and $s_2(t) = 0$ for all $t$. Is this a Nash equilibrium?

*b)* Find a stationary symmetric Nash equilibrium, where both players stop with the same constant probability in each period.

*c)* Are the strategy profiles considered above subgame perfect equilibria?

*d)* Can you think of other strategy profiles that would constitute a subgame perfect equilibrium?

SPE is our "default" solution concept for multi-stage games, therefore it is important to carefully and critically examine it. In problem 3 we compare NE and SPE outcomes of the simple games with their actual play in experimental setting [based on Goeree and Holt (2001)].

**Problem 3 (Experimental evidence on SPE)**

*a)* Consider the extensive form of the game in Figure 1. What are NE and SPE of the game? In the experimental setting, $16\%$ of randomly matched pairs played the game with the outcome of $(80, 50)$, and the rest played the game with the outcome of $(90, 70)$. How do you interpret this finding, as in what does it tell us about SPE as a solution concept?

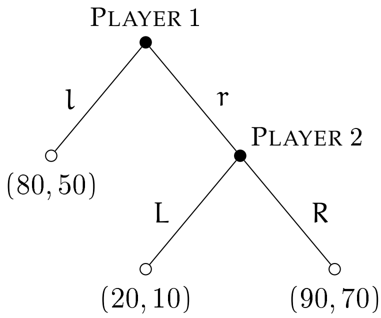


Figure 1: Should you trust others to be rational?

*b)* Consider the extensive form of the game in Figure 2. What are NE and SPE of the game? How do you think empirical distribution of outcomes changes compared to the game in *a)* and why?

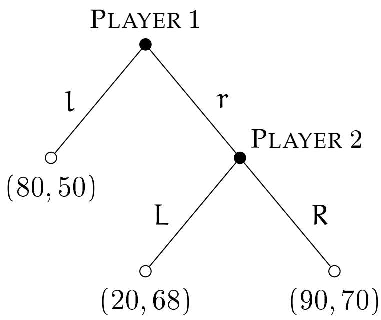


Figure 2: Revisited “Should you trust others to be rational?"

*c)* Consider the extensive form of the game in Figure 3. What are NE and SPE of the game? In the experimental setting, $12\%$ of randomly matched pairs played the game with the outcome $(70, 60)$, and the rest played the game with the outcome of $(90, 50)$. How do you interpret this finding?

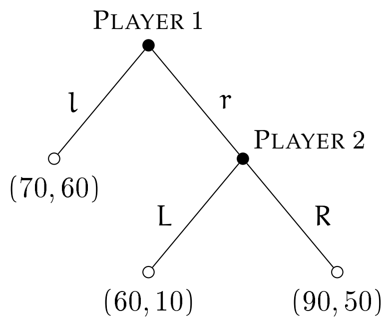


Figure 3: Should you believe a threat which is not credible?

*d)* Consider the extensive form of the game in Figure 4. What are NE and SPE of the game? In the experimental setting, $32\%$ of randomly matched pairs played the game with the outcome $(70, 60)$, $32\%$ played the game with the outcome of $(60, 48)$ and the rest played the game with the outcome of $(90, 50)$. Does this empirical distribution support SPE? Why?

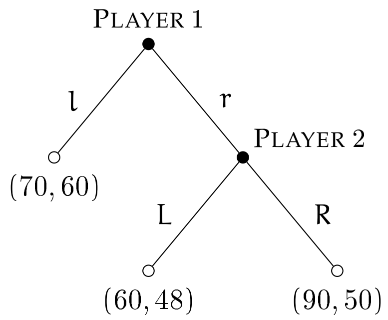


Figure 4: Revisited "Should you believe a threat which is not credible?"

Problem 4 below demonstrates issues with using SPE for analysis of dynamic games with incomplete information.

**Problem 4 (SPE in games with incomplete information)**

*a)* What are SPE of the game in Figure 5? Are all of them sequentially rational?

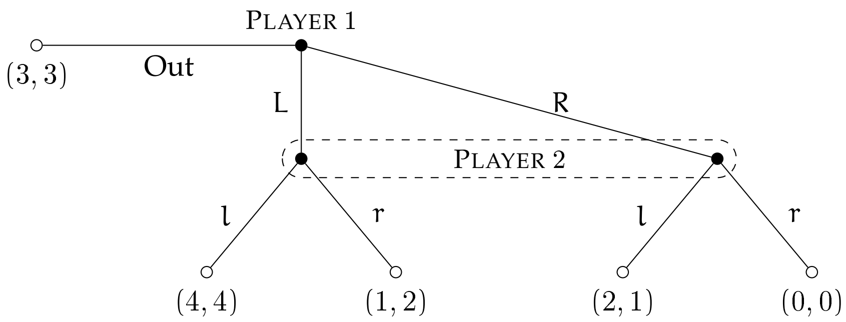


Figure 5: Deadlock game with an outside option

*b)* In the game of Figure 6, Nature chooses L with probability $\frac{3}{4}$ and R with probability $\frac{1}{4}$. What are SPE of the game?

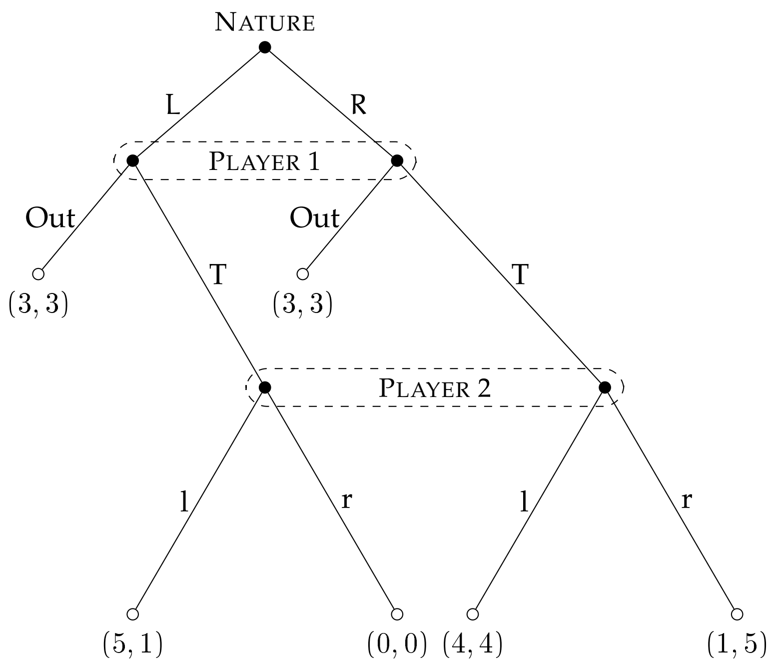


Figure 6: SPE supported by inconsistent beliefs

**Problem 5 (Perfect Bayesian Equilibrium)** *What are Perfect Bayesian Equilibria of the games in problem* 4*?*

## References

1. Jacob K. Goeree, Charles A. Holt. "The little treasures of game theory and ten intuitive contradictions". AER, 2001.